Suggested Solutions to: Resit Exam, Spring 2016 Industrial Organization August 11, 2016

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Question 1: Dynamic monopoly and the Coase conjecture (the two-period "movie model")

To the external examiner: The students had seen this model before. It was discussed in a lecture. An extension of the model (with three periods) was also discussed in an exercise class.

Part (a)

- We can solve the model by first studying the optimal behavior in period 2 for the firm and the consumers, given some arbitrary cut-off point $\widehat{r} \in (0,1)$. Then, after having found the equilibrium value of p_2 as a function of $\widehat{r}$, we can study the optimal behavior in period 1, thereby identifying the equilibrium values of $\widehat{r}$ and p_1.
- Remember that the monopoly firm is myopic — it cares only about the current period's profit when choosing the current period's price. The consumers, however, care about their future utilities — they use the (common) discount factor δ.

Second period

- Suppose consumers with $r > \widehat{r}$, for some $\widehat{r} \in (0,1)$, consume in period 1.
  - The variable $\widehat{r}$ is of course endogenous and we will later on determine its equilibrium value (in terms of exogenous parameters).
- In period 2, the monopolist then faces the demand schedule

$$q_2 = \widehat{r} - p_2.$$

The derivation of this demand function makes use of the assumption that the r's are uniformly distributed on $[0,1]$ and the fact that the remaining consumers in period 2 buy if and only if their valuation $r \in [0,\widehat{r}]$ exceeds the price p_2. (The students may want to draw a figure to illustrate how the demand function is obtained.)

- The price that maximizes period 2 profits, $\pi_2 = (\widehat{r} - p_2)\,p_2$, is

$$p_2 = \frac{\widehat{r}}{2}. \tag{1}$$

First period

- Given the period 1 price p_1 and the period 2 price $p_2 = \frac{\widehat{r}}{2}$, a consumer will consume in period 1 if and only if

$$r - p_1 \geq \delta\,(r - p_2) = \delta\left(r - \frac{\widehat{r}}{2}\right). \tag{2}$$

Remember that $\widehat{r}$ is defined as the value of r that makes the above inequality hold with equality:

$$\widehat{r} - p_1 = \delta\left(\widehat{r} - \frac{\widehat{r}}{2}\right) \Leftrightarrow \widehat{r} = \frac{2p_1}{2-\delta}. \tag{3}$$

- The firm's profit at the stage when it chooses the period 1 price:

$$\pi_1 = [1 - \widehat{r}]\,p_1 = \left[1 - \frac{2p_1}{2-\delta}\right]p_1.$$

- FOC:

$$\frac{\partial \pi_1}{\partial p_1} = 1 - \frac{4p_1}{2-\delta} = 0$$

or

$$p_1^* = \frac{2-\delta}{4}. \tag{4}$$

Summing up

- By plugging (4) into (3), we can now get the equilibrium cut-off point

$$\widehat{r}^* = \frac{2p_1^*}{2-\delta} = \frac{1}{2}. \tag{5}$$

- By plugging (5) into (1), we get the equilibrium period 2 price

$$p_2^* = \frac{\widehat{r}^*}{2} = \frac{1}{4}.$$

- At the equilibrium we thus have

$$\boxed{p_1^* = \tfrac{2-\delta}{4} \qquad \text{and} \qquad p_2^* = \tfrac{1}{4},}$$

and half of the consumers consume in the first period ($\boxed{\widehat{r}^* = \tfrac{1}{2}}$).

Part (b)

- The Coase conjecture concerns a situation where a monopoly firm, in each one of many periods, sells a good that is durable. The firm is allowed to choose a new price in each period. The fact that the good is durable means that those costumers who have bought the good will not need to purchase the good in any future period — these customers disappear from the demand. The Coase conjecture (it was later proven to, under certain conditions, hold as a result) states that:

  - When the length between time periods become smaller (or, equivalently, when the consumers' discount factor approaches one), the monopolist's price converges to the marginal cost — the firm loses all its market power.

- The reason why this happens is that for any given price in a period, the consumers who find it worthwhile to purchase will be those with the highest valuation. That means that in the next period, those high-valuation consumers are not part of demand and therefore the optimal monopoly price must be lower (since demand is lower). In other words, if the monopoly firm cannot precommit to some sequence of prices but is optimizing in each period given the current demand, the price will gradually drop. However, if the consumers understand this they should have an incentive to wait with purchasing until a later period when the price has fallen. The only thing that may stop the consumers from waiting is that they are impatient and prefer immediate consumption to later, all else being equal. But if the length of time between periods is small or if the consumers are not very impatient (which is the condition in the conjecture), then the consumers don't mind waiting until the price has dropped. If so, the firm may be better off lowering the price straight ahead, so that it doesn't have to wait so long for its (perhaps small) profits.

- To further clarify the explanation we can relate to the result we obtained under a). In that model, whereas the second-period price is constant, *the first-period price is decreasing in the patience parameter δ.* This result is in the spirit of the Coase conjecture, although the monopolist in this simple example doesn't lose all its market power, only some of it.

Part (c)

The Herfindahl index is defined as the sum of the squared market shares, $HI = \sum_{1=1}^{n} s_i$, where s_i is firm i's market share and n is the number of firms in the market.

- Therefore, the Herfindahl index for this market equals

$$\begin{aligned} HI = 2 \times \left(\frac{5}{100}\right)^2 + 2 \times \left(\frac{10}{100}\right)^2 \\ +2 \times \left(\frac{20}{100}\right)^2 + \left(\frac{30}{100}\right)^2 \\ = \frac{50}{10,000} + \frac{200}{10,000} + \frac{800}{10,000} + \frac{900}{10,000} = \frac{1,950}{10,000} \\ = 0.195. \end{aligned}$$

The 3-firm concentration index ratio is defined as the sum of the three largest firms' market shares.

- Therefore this ratio equals $0.3+0.2+0.2 = 0.7$.

Question 2: Collusion with fluctuating and persistent demand

To the external examiner: The students had not seen this exact model before, but it is of course based on material that they have seen in the course. First, the students have studied the Rotemberg-Saloner model, which concerns the same question as here (but with time-independent states). Second, they have studied the Green-Porter model, which uses the methodology that is required here.

Part (a)

- Equations (1) and (2) are two equalities that both must hold by definition.
  - The term V_H is the expected present-discounted stream of profits, given that the firms collude, that is earned by a firm that knows that the present period's state is *high*.
  - Similarly, the term V_L is the expected present-discounted stream of profits, given that the firms collude, that is earned by a firm that knows that the present period's state is *low*.
- The right-hand side of equation (1) is another way of writing V_H, in terms of itself and V_L.
  - The first term is half of the high-state industry profits that the firms earn jointly when colluding.
  - The second term, $\delta[\alpha V_H + (1-\alpha)V_L[$, is the expected present-discounted stream of profits that the firm will start earning in the following period, when continuing to collude, but discounted with the factor δ (since the profits are evaluated from the perspective of the present period). With probability α the state is high, and then the firm is in the same situation as it was at the outset, and thus faces the expected present-discounted stream of profits V_H. With probability $1-\alpha$ the state is low; then the firm is in the same situation as it was at the outset, but with a low instead of a high state, and it thus faces the expected present-discounted stream of profits V_L.
- The right-hand side of equation (2) is another way of writing V_L, in terms of itself and V_H. The logic is very much as above, for equation (1) — but note that here β plays the role of $1-\alpha$.
- Let us now state the two Nash conditions (on the equilibrium path). First consider a firm's incentive to deviate in a situation where it knows that the state is high. If following the equilibrium strategy when the state is high, the firm earns V_H. If the firm instead makes the best possible deviation (slightly undercutting the rival), then it can earn (almost) the full industry profit in the current period, and after that zero (since the deviation will lead to Bertrand competition and zero profits). The firm thus does not have an incentive to deviate if, and only if, the following condition holds:
$$V_H \geq \pi_H^m. \tag{6}$$
- Similarly, the Nash condition for the low state is
$$V_L \geq \pi_L^m. \tag{7}$$

Part (b)

The reason is that in a high-demand state demand will be *unusually* high. The demand realization is by assumption independent over time, so the expected profits tomorrow and onwards are the same regardless of today's demand state. This means that when the demand is known to be high today, then the incentive to deviate from the equilibrium is higher than in the standard model, as the "one-period temptation" is unusually high whereas the "long-term reward of not deviating" is the same. The conclusion is that there is a tendency for collusion to break down in a high-demand state (hence price war during booms and counter-cyclical prices).

Part (c)

The Nash condition in (6) can be written as

$$\begin{aligned} V_H \quad &\geq \quad \pi_H^m \Leftrightarrow \frac{3\,(1-\delta\alpha)+2\delta(1-\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]} \geq 6 \\ &\Leftrightarrow \quad \frac{6\,(1-\delta\alpha)+4\delta(1-\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]} \geq 12. \end{aligned}$$

And the Nash condition in (7) can be written as

$$\begin{aligned} V_L \quad &\geq \quad \pi_L^m \Leftrightarrow \frac{3\delta\,(1-\alpha)+2(1-\delta\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]} \geq 4 \\ &\Leftrightarrow \quad \frac{9\delta\,(1-\alpha)+6(1-\delta\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]} \geq 12. \end{aligned}$$

The first Nash condition is thus strictly more stringent if

$$\frac{6\,(1-\delta\alpha)+4\delta(1-\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]} < \frac{9\delta\,(1-\alpha)+6(1-\delta\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]}$$

$$\begin{aligned}\Leftrightarrow 6\,(1-\delta\alpha)+4\delta(1-\alpha) &< 9\delta\,(1-\alpha)+6(1-\delta\alpha)\\ \Leftrightarrow 4\delta(1-\alpha) &< 9\delta\,(1-\alpha)\\ \Leftrightarrow 4 &< 9,\end{aligned}$$

which always holds.

We can conclude that collusion is most difficult to sustain in a high state.

Part (d)

We found in part (c) that the most stringent Nash condition is the one for the high state. This condition is relaxed by an increase in α if, and only if, the left-hand side (i.e, V_H) is increasing in α. Differentiating yields

$$\begin{aligned} & \frac{\partial}{\partial\alpha}\left[\frac{3\,(1-\delta\alpha)+2\delta(1-\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]}\right]\\ = & \frac{-5\delta[1-\delta(2\alpha-1)]+2\delta\,[3\,(1-\delta\alpha)+2\delta(1-\alpha)]}{(1-\delta)[1-\delta(2\alpha-1)]^2}\\ = & \frac{\delta\,\{-5\,(1-\delta-2\delta\alpha)+2\,(3+2\delta-5\delta\alpha)\}}{(1-\delta)[1-\delta(2\alpha-1)]^2}\\ = & \frac{\delta\,(1+9\delta)}{(1-\delta)[1-\delta(2\alpha-1)]^2} > 0,\end{aligned}$$

which confirms that the left-hand side is indeed increasing in α.

We can conclude that a higher value of α makes collusion easier.